

DM545/DM871 – Linear and integer programming

Exercise Sheet 6, Autumn 2026

Exercises with the symbol $+$ are to be done at home before the class. Exercises with the symbol $*$ will be tackled in class and should be at least read at home. The remaining exercises are left for self training after the exercise class.

Exercise 1* Sensitivity Analysis and Revised Simplex

We reconsider the problem of the furniture-manufacturing company, which we solved in the previous sheet. For completeness we report here again the introductory description.

A furniture-manufacturing company can produce four types of products using three resources.

- A bookcase requires three hours of work, one unit of metal, and four units of wood and it brings in a net profit of 19 Euro.
- A desk requires two hours of work, one unit of metal and three units of wood, and it brings in a net profit of 13 Euro.
- A chair requires one hour of work, one unit of metal and three units of wood and it brings in a net profit of 12 Euro.
- A bedframe requires two hours of work, one unit of metal, and four units of wood and it brings in a net profit of 17 Euro.
- Only 225 hours of labor, 117 units of metal and 420 units of wood are available per day.

In order to decide how much to make of each product so as to maximize the total profit, the managers solve an LP problem.

- 1) Write the mathematical programming formulation of the problem.
- 2) With the help of a computational environment such as Python, using only matrix operations, carry out the first iteration of the revised simplex method.
- 3) determine the value of the solution that has in basis the variables x_1 , x_3 and x_4 ,
- 4) determine the value of the dual variables associated with the constraints,
- 5) verify that the solution is indeed optimal.

Exercise 2 Factory Planning and Machine Maintenance

Tasks 1-2 of [Factory Planning and Maintenance Case](#).

Exercise 3 Crop Planning

The following are two cases of crop planning that arise in agriculture. They are two separated cases and do not share information. The first case considers the single period scenario while the second a multi-period scenario. In real-life applications, a combination of the two cases is the most likely needed.

Subtask 3.1

Increasing the efficiency in agriculture by exploring the possibilities of optimal allocation of the scarce resources for the various farms has become a urgent need of society. Crop planning is one of the important stages in crop production and it has a considerable effect on farmer revenue.

Assume we have collected data regarding land allocation, labor days, machine hours, cost of seeds, fertilizers, and pesticides from farmers for four crops: paddy, cotton, chilli and maize, which are grown in a region.

Using averages of these data we want to determine the crop plan that maximizes the net return for an average farmer of a given region. The profit that farmers make per hectare ($1 \text{ ha} = 10.000 \text{ m}^2$) of arable area of the four crops, paddy, cotton, chilli and maize are in Danish krone: 15.434, 46.186, 160.607, and 14.088, respectively.

The resource constraints to be considered are land, labor in man days, machine hours, operating cost and alternating crop plans.

- **Land.** Land is the basic resource of production. All land related activities are expressed in hectares basis. The total land available that we consider is 46 hectares and the total arable area cannot exceed this size.
- **Labor.** The labor is taken in man days. The man days needed per hectare of cultivation of the four crops are 29, 58, 308 and 24, respectively and the capacity is 5819 days.
- **Machine hours.** Farmers are hiring the tractor services for land preparation and machine hours are limited. The machine hours required per hectare of area of the four crops are 5, 4, 2, and 4, respectively, and the capacity is 525 hours.
- **Operating capital.** The operating capital refers to the funds required to meet the cost of seeds, fertilizers, farm yard manure, plant protection chemicals, insurance charges, marketing expenses and wages of the human and bullock labor and tractor power. The operating costs per hectare of area of the four crops are 32.474, 38.897, 111.962, 21.713, respectively, and the maximum budget is 4.343.868 Dkk.
- **Alternate crop plans.** The alternate crop plans are developed by introducing minimum and maximum land allocation constraints for the crops because in practical situations it is not applicable to allocate the whole land to one crop due to the fluctuations in the demand and risk associated with specialization of one crop. Here, we consider only a limitation on the crop that is cultivated by the majority of the farmers, paddy. The portion of (total available) land that can be allocated to paddy crop must be reduced by 20%.

Formulate the problem as a linear programming problem.

Subtask 3.2

Common Agricultural Policies stipulate a “payment for agricultural practices which are beneficial for the climate and the environment”. This is commonly known as “greening payment” and represents a direct aid per hectare rewarding agricultural sustainable practices.

The greening payment may potentially encourage farmers to meet certain environmental requirements in return for governmental support payments. This greening aid rewards farmers complying three basic “greening rules” (or equivalent practices). These are: (1) maintenance of existing permanent grassland (2) establishment of an ecological focus area on arable land and (3) crop diversification.

We consider a farmer who needs to decide whether to adapt his/her cropping plan in order to fulfill the greening rules and therefore receive the greening subsidy. The farmer seeks to maximize his/her revenues over the time horizon of the policy, that is, the farmer seeks to optimize cropping plan decisions for the next five years by maximizing his/her net return.

Let us consider a farmer that owns a piece of arable land of 35 hectares (ha) where $N = 6$ different crops may be grown. We consider a planning period of $T = 5$ years, with one growing season per year. We are given the crop yields, e_i , and the cost of exploitation, c_i , of each crop $i = 1..6$. Moreover, the prices p_i received by farmers for each crop and the value of the greening subsidy payment b_i for each crop are available. These data are shown in Figure 1. The first table above shows the crops considered, their selling price and their exploitation cost, the other two tables below show the yield and the basic

payment per crop type. All this data is per unit of hectare and we assume a linear growth with respect to cultivated hectares for the corresponding crop.

The price received by the farmer and the exploitation cost of the crops are capitalized each year by recursively updating the current price and cost of the corresponding crop (p_i and c_i , respectively) at a given inflation rate (i.e., consumer price index) $r = 5\%$:

$$p_{it} = p_i \cdot (1 + r)^t \quad i = 1..N, t = 1..T$$

$$c_{it} = c_i \cdot (1 + r)^t \quad i = 1..N, t = 1..T$$

The farmer's net return NV_t (€) at the end of period t must be actualized to the *net present value* at $t = 0$. The total net present value accumulated per hectare of each crop $i = 1..N$ during the planning horizon can be derived by means of the following formula:

$$NPV_i = \sum_{t=1}^T \frac{(e_i p_{it} + b_i - c_{it})}{(1 + a)^t}$$

where a is a discount rate of 5%.

The goal is to determine the optimal cropping plan by choosing the crops to be grown every year and their acreage in order to maximize the total farmer's economic net present value over the time horizon of five years.

The greening subsidy b_i is however only received if the three basic greening rules of the policy are fulfilled: (1) maintenance of permanent grassland (2) establishment of ecological focus areas and (3) crop diversification.

More precisely, the restrictions imposed by the greening rule are as follows:

- Arable land. The land allocated to the crops, the permanent grassland area and the ecological focus area must not exceed the total farm arable land each year.
- Maintenance of permanent grassland. The permanent grassland has to be at least 5% of the total arable land every year.
- Establishment of ecological focus areas. The ecological focus area has to be greater than 5% of the total arable land in the previous year up to year $t = 2$, and greater than 7% of the previous year from $t = 3$.
- Crop diversification. The farmer must grow at least three different crops. The main crop (i.e., the crop occupying the largest area) must not cover more than 75% of the total arable land in each period t . The two largest crops together must not cover more than 95% of the arable land.
- Crop rotations. The same crop cannot be grown in the same area in sequential seasons. To allow this it must be enforced that the arable land occupied by one crop in period t is lower or equal to the surface not occupied by this crop during the previous period. For the first planning period we assume the area occupied by the crops in the previous year was 0 ha.

A mathematical model of the problem and its solution can provide valuable insight to both farmers and Governments: farmers can optimize their revenue and Governments can determine the minimum subsidy value that would make the implementation of greening rules economically appealing (eg, by comparing farmer's revenue with and without the greening rules).

Formulate the problem as a linear programming problem.

<i>i</i>	Crop	Price (€t ⁻¹)	Cost (€ha ⁻¹)
1	Wheat	230.1	604.1
2	Barley	194.8	598.1
3	Rye	181.5	377.6
4	Oat	183.6	585.7
5	Dried Pea	223.6	455.6
6	Sunflower	380.1	430.9

Yield (t ha ⁻¹)						Basic payment (€ha ⁻¹)					
Wheat	Barley	Rye	Oat	Dried Pea	Sunflower	Wheat	Barley	Oat	Rye	Dried peas	Sunflower
2.396	2.334	1.468	1.642	0.811	0.774	150.96	147.05	92.46	103.45	181.00	48.73

Figure 1: Above: Crops considered, selling prices and production costs per crops. Below left: Crop yields. Below right: Basic payment for crops.