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DM545/DM871 – Linear and integer programming

Exercise Sheet 8, Autumn 2026

Exercises with the symbol $^+$ are to be done at home before the class. Exercises with the symbol * will be tackled in class and should be at least read at home. The remaining exercises are left for self training after the exercise class.

Exercise 1$^+$

Show that the maximum matching in arbitrary graphs is a special sort of set packing problem.

Exercise 2 Bin Packing

The binpacking problem is to assign a given set of $n \in \mathbb{N}$ items, each of a size $s_i \in \mathbb{N}$, to bins with capacity $\kappa \in \mathbb{N}$, such that the used bins are not overloaded. The goal is to minimize the number of used bins. Model the problem as a MILP problem.
Example: Given 9 items with sizes: 2, 1, 2, 1, 1, 2, 3, 2 and 1, and a bin capacity of $\kappa = 4$. Figure 1 shows a feasible solution which needs 5 bins.

Exercise 3*

Give two MILP formulations for the chromatic number problem: given a graph find a coloring (i.e., a mapping from the natural numbers to the set of vertices) such that every pair of vertices connected by an edge receives different colors and the number of colors used is the least possible.

Exercise 4*

Formulate the Minimum Spanning Tree problem as a mathematical programming problem.

Exercise 5* Consider the polyhedron $P \subseteq R^2$ described by the following inequalities and depicted in Figure 2:

$$\begin{array}{rcrcr} 2x & - & y & \leq & 4 \\ 2x & + & 3y & \leq & 12 \\ & & y & \leq & 3 \\ 3x & + & 2y & \geq & 6 \\ x & & & \leq & 3 \\ 3/2x & + & y & \leq & 45/8 \\ 2x & + & 3y & \leq & 10 \end{array}$$

Which inequality is a face? Which is a facet? Which is redundant?

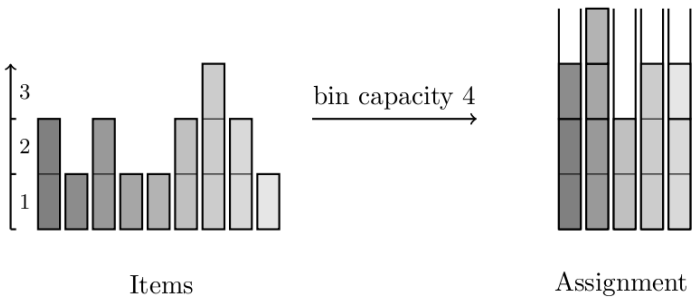


Figure 1: A feasible solution to the bin packing problem.

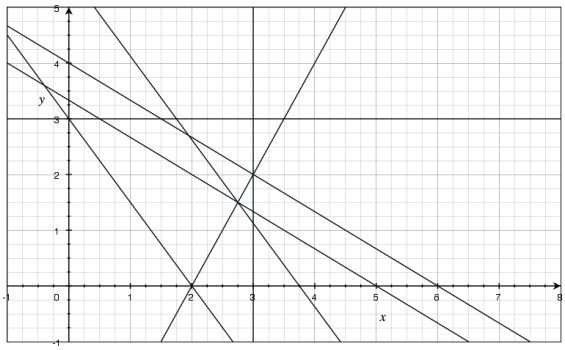


Figure 2: The polyhedron of exercise 5.

Exercise 6*

Model:

- the vertex covering problem: given a graph find the set of vertices such that each edge of the graph is incident to at least one vertex of the set.
- the dominating set problem: given a graph find a set of the vertices such that every vertex not in the set is adjacent to at least one member of the set.

Are the solutions to the two problems always the same, i.e., are the two problems equivalent?
The two problems have several applications, for example: in wireless networking, dominating sets are used to find efficient routes within ad-hoc mobile networks; they also encode the art gallery problem or museum problem, that is, guarding an art gallery with the minimum number of guards who together can observe the whole gallery.

Exercise 7*

Given a collection of sets, a set which intersects all sets in the collection in at least one element is called a *hitting set*, and a simple NP-hard problem is to find a hitting set of smallest size or minimum hitting set. Formulate this problem in MILP terms.

Exercise 8

Four friends are going on holiday together and need to plan out a schedule of activities for the next two weeks. Each day has three time slots, in which an activity can be placed, representing morning, midday and evening. The goal is to schedule as many different activities as possible. Each activity can either be relaxing, active or cultural (denoted by r, a and u). Associated with each activity is a cost. The total cost for the two weeks must not exceed 14.000kr. All slots should be assigned to exactly one activity. Because they do not want to go overtired, each day can at most have one active activity and it cannot be in the morning. They would also like to reserve the first day for only cultural and relaxing activities. The three last days may not exceed 700kr for each day, because they want to save money for souvenirs. They do not want to do the same cultural or active activities twice, but would accept doing the same relaxing activities three times. Further, each person has a list of activities they would like to add, so atleast one activity from each friend's list must be scheduled.

Exercise 9

The Classical airline crew scheduling problem is described as follows. A flight segment is a nonstop flight between two airports with particular departure/arrival times and airplane requirements. Crew schedules are determined for each fleet (airplanes of a particular type) separately. First a subset of *rotations (or rosters)* are identified. Rotations are collections of flight segments that satisfy aviation regulations, company regulations and labor union rules and that are meant to be served by a single crew. They have typically duration of 2-3 days. Then, crews are assigned to the selected rotations, often by a bidding system.

The cost of selecting a particular rotation includes flight time of the crew in that rotation, lodging and per day expenses, cost of transporting crews to their next departure city in their next flight segment (they are not serving on these flight segments), etc. Let c_j denote the cost of selecting rotation $j \in \{1, \ldots, n\}$. Suppose there are m flight segments, then a rotation j is given by an incidence vector of flight segments in that rotation, i.e., for rotation j, $\mathbf{a}_j$ is an m-dimensional vector with $a_{ij} = 1$ if flight segment i is on rotation j and 0 otherwise. Formulate the problem of selecting the rotations that minimize the total cost such that all flight segments are served, and each flight segment is served by exactly one rotation.